Technical Comments

Comment on "Use of Ion Probes in Supersonic Plasma Flow"

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IN a recent paper[1] Scharfman and Taylor present measurements of ionization relaxation times for air in the Mach number range from $M_s = 8$–$M_s = 34$. They find that their results are in good agreement with data which have been obtained in previous papers by Wilson,[2] Lin, Neal and Fyfe[3] and by Frohn and de Boer.[4,5] The authors of this Comment published measurements for Mach numbers between 8 and 17 in the proceedings of the Ninth International Conference on Phenomena in Ionized Gases 1969 in Bucharest.[6] These data which are not included in Scharfman's paper are also in good agreement with the calculations by Thompson as can be seen in Fig. 1.

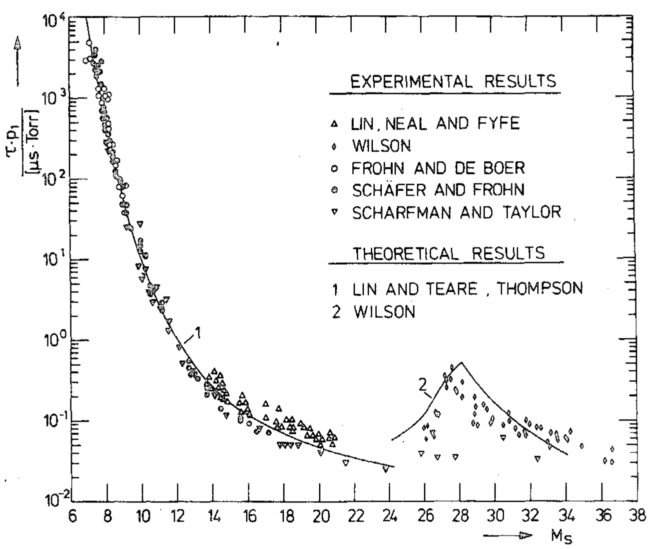

Fig. 1 Product of relaxation time τ and initial pressure p_1 as function of shock Mach number M_s.

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[6] Schäfer, J. H. and Frohn, A., "Shock Tube Measurements of Ionization Relaxation Time in Air," *International Conference on Phenomena in Ionized Gases, Ninth, Bucharest, Romania*, September 1–6, 1969, edited by Institute of Physics, Academy of the Socialist Republic of Romania, p. 53; also *International Aerospace Abstracts*, Feb. 1, 1970, Vol. 10, No. 3, Accession No. A. 70-14360, p. 471.

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Comment on "Analysis of an Active Thermal Protection System for High-Altitude Flight"

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IN Ref. 1 Libby and Hendricks consider a transpiration cooling system in which the coolant required for protecting the nose, or leading edge regions, is obtained by suction of the boundary layer in downstream regions. The claim is made that "with appropriate distribution of injection and suction there can be designed a thermal protection system which is adiabatic in the sense that no internal refrigeration is required and which consumes no coolant." The authors proceed to present analyses relating to the determination of suitable mass transfer distributions. The purpose of this comment is to show that, due to an unfortunate error, the systems analysed do require internal refrigeration; the results presented in Ref. 1 do not apply to adiabatic systems. In fact, the optimum suction distribution for an adiabatic system may well have to satisfy criteria quite different to those imposed in Ref. 1.

Equation (1) of Ref. 1 is an energy balance on an element of porous wall with convective heating and radiative cooling on the exposed surface. It may be rewritten as

$$\rho v(h_w - h_c) = q_{\text{conv}} - q_{\text{rad}} \tag{1}$$

where h_c is the enthalpy in the coolant chamber. In the injection and suction regions, the coolant enthalpy is prescribed to be $h_{c,1}$ and $h_{c,2}$, respectively, with $h_{c,1} > h_{c,2}$. In addition, the authors prescribe h_w and q_{rad} by choosing a practical temperature limit for the porous material. The error in the analyses of Ref. 1 lies in the authors' belief that they can prescribe $h_{c,2} < h_w$ for a system which does not have internal refrigeration. We shall show that, in the absence of refrigeration, $h_{c,2} = h_w$. Thus the mass transfer distributions obtained in Ref. 1, where $h_{c,2}$ was set equal to 0.5 h_w, will not achieve the authors' objective of an "adiabatic" system.

The claim of $h_{c,2} = h_w$ should be quite acceptable on simple physical grounds. But in order to remove all doubts a mathematical demonstration appears necessary. Thus we shall indicate the correct solution to the problem posed by Libby in Ref. 2. Using the notation and coordinate system of Refs. 2 and 3, the temperature distributions in the wall and gas are governed by

$$\lambda_s T'' = h(T - t) \quad 0 \leq x \leq L \tag{2}$$

$$\rho v c_p t' - \lambda_g t'' = h(T - t) \quad 0 \leq x \leq L \tag{3}$$

$$\rho v c_p t' - \lambda_g t'' = 0 \quad L \leq x < \infty \tag{4}$$

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In Ref. 2, Libby did not write down Eq. (4), but instead asserted "it is readily clear that for $x > L$, $t \equiv t_c$" and proceeded to apply $t(L) = t_c$ as a boundary condition on Eq. (3). At this point we stress that Eq. (4) is a necessary part of the mathematical description of the problem posed by Libby in Ref. 2; inclusion of gas conduction requires that the solutions in the two regions, $0 \leq x \leq L$ and $L \leq x < \infty$, be correctly matched. The solution of Eq. (4) is

$$t = C_1 \exp[(\rho v c_p/\lambda_g)x] + C_2 \quad L \leq x < \infty \tag{5}$$

For suction v is positive and Eq. (5) shows that t grows exponentially with x. Thus it is not possible to impose a Dirichlet type boundary condition as $x \to \infty$, which is what Libby essentially did by setting $t \equiv t_c$. Instead the correct boundary conditions on Eq. (4) are

$$x = L: \qquad t(L^+) = t(L^-) = t(L)$$

$$x \to \infty: \qquad t' = 0 \text{ (no internal refrigeration)}$$

and the correct solution of Eq. (4) is therefore

$$t \equiv t(L) \tag{6}$$

where $t(L)$ is as yet still to be determined. The additional four boundary conditions required for the sixth order mathematical system are

$$x = 0: \qquad -\lambda_s T' - \lambda_g t' = q_{\text{conv}} - q_{\text{rad}} \tag{7}$$

$$T = t \tag{8}$$

$$x = \text{L}: \qquad -\lambda_s T' = 0 \tag{9}$$

$$-\lambda_g t(L^-) = 0 \text{ [since } t'(L^+) = 0] \tag{10}$$

Solution of Eqs. (2) and (3) subject to these boundary conditions yields the trivial solution

$$T = t = \text{const} = T_w \tag{11}$$

and T_w is determined by Eq. (7) which degenerates to

$$0 = q_{\text{conv}} - q_{\text{rad}} \tag{12}$$

Thus we have shown that $t_c = t_w$ (or $h_c = h_w$) is the correct solution of the suction problem. We note in passing that in Ref. 2, Libby went further astray by not recognizing that Eq. (9) must hold, i.e., the physical requirement that the heat flux in the solid at $x = L$ must be zero, unless for example, cooling coils are brazed onto the backface of the wall.

Returning to our criticism of Ref. 1, where solutions were obtained for prescribed $h_{c,2} < h_w$, it is clear that such systems will require internal refrigeration. This additional thermodynamic burden is contrary to the authors' intent. Furthermore, an adiabatic system will require the suction region value of h_w to be lower than its prescribed value in the injection region. Thus the convenient simplifications which result from the assumption of position independent h_w and q_{rad}, and which were exploited by Libby and Hendricks, would not apply to an adiabatic system. Finally, although ingested air is a possible source of transpiration coolant, it has yet to be shown that the scheme considered in Ref. 1 has adequate potential for the proposed applications.

References

[1] Libby, P. A. and Hendricks, P., "Analysis of an Active Thermal Protection System for High Altitude Flight," *AIAA Journal*, Vol. 8, No. 9, Sept. 1970, pp. 1671–1678.

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Reply by Author to A. F. Mills

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For the reader who appreciates clarity and lacks the time to assess for himself a technical controversy, it is regrettable that Professor Mills did not avail himself of the opportunity we afforded him to comment prior to submittal on our rejoinder to his first criticism. If he had done so, we could perhaps have developed a single response and avoided his present comment.

Professor Mills' objection can be succinctly stated; he requires that both conduction terms, the one in the solid, $\lambda_s T'(L^-)$, and the one in the gas $\lambda_g t'(L^-)$, be individually zero whereas we require only that their algebraic sum be zero. Despite Professor Mills' present argument we see no physical or mathematical reason to impose this extra requirement and believe our analysis to be correct. Thus the reader must expend some time if he wants clarification of this matter.

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